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O. E. Aleksandrov<sup>a</sup>

<sup>a</sup> Department of Technical Physics , Ural Federal University (UPI) , Ekaterinburg , Russian Federation

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# The Upper Limit of the Separation Efficiency of a Gas Centrifuge

O. E. Aleksandrov

Department of Technical Physics, Ural Federal University (UPI), Ekaterinburg, Russian Federation

A general theory of isotopes separation in a gas centrifuge based on a radial averaging method has been developed. The ultimate upper limit for a centrifuge separative power is expressed as a function of its external parameters. This is a more accurate definition of the centrifuge efficiency upper limit than the well-known Dirac's estimation or estimations of his followers, because it takes into account the feed flow value (throughput) and it provides the way to a one-dimensional diffusion equation without a lot of assumptions. For the first time the problem of an energy efficient centrifuge is formulated and the solution is obtained. Two cases of a centrifuge internal flow optimization are compared. The optimal internal circulations for both optimization cases are calculated. The results help us to understand how far the modern centrifuges are from their highest possible effectiveness limit and to identify ways to improve centrifuge performance.

**Keywords** centrifugation; convective transport; diffusion; gases; mathematical modelling; optimization

## INTRODUCTION

To develop an effective gas centrifuge (GC) for isotope separation, it is necessary to have an accurate estimate of the centrifuge productivity reserves. At present, the main efforts to describe GCs are directed at developing simulation programs for an internal gas flow, when the exact limit of GC efficiency still remains unknown. This paper approaches the problem from a different point—by calculating the optimal gas flow in the GC rotor in order to obtain the highest separation efficiency.

The GC efficiency is expressed in units of the separative power:

$$\delta U = P \cdot V(N_p) + W \cdot V(N_w) - F \cdot V(N_F), \quad (1)$$

where  $\delta U$  is the separative power;  $F$ ,  $P$ , and  $W$  are the feed (throughput), product, and waste flows of a GC, respectively;  $N_F$ ,  $N_p$ , and  $N_w$  are the mass concentration in the

feed, product, and waste flows, respectively;  $V$  is the value function

$$V(N) = (2N - 1) \ln \frac{N}{1 - N}.$$

The separative power of a GC depends on many factors. One important factor is the internal gas flow within the GC rotor (Fig. 1). The classical estimate of the separative power maximum according to Dirac (1) is:

$$\delta U_D = \frac{\pi}{2} H \rho D \left( \frac{\Delta m \cdot (R\Omega)^2}{2kT} \right)^2 \quad (2)$$

where  $H$  is the GC rotor length,  $D$  is the diffusion coefficient,  $\rho$  is the density of the gas being separated,  $\Delta m$  is the difference between the molecular masses of the isotopes ( $\Delta m = |m_1 - m_2|$ ),  $R$  is the inner radius of the rotor,  $\Omega$  is the rotor angular rotation speed,  $k$  is Boltzmann's constant, and  $T$  is the gas temperature.

Any attempts to achieve GC efficiency comparable with the estimate (2) have so far been futile. No connection between the GC separative power and the external and internal flows has been considered in the estimate. This makes the estimate applicable only to a single operation regime, viz., one of infinitely large feed and circulation flows in the rotor. A centrifuge is unable to operate in such a regime. The GC flow parameters are chosen as a compromise between the requirements of the circulation stability within the rotor and a finite value of the separation factor. Thus, the finding of a set of parameters that ensure maximum efficiency of a GC is of great importance.

There are also other inconsistencies between an experiment and Eq. (2). For example, the experimental dependence of  $\delta U$  on the GC rotor length is nonlinear. Till the latest publications (2) the researchers could not exactly predict the  $\delta U_{max}$  dependence on the centrifuge rotor length. The previous works (1,3) introduced a "flow efficiency" coefficients to describe those effects. The "flow efficiency" coefficients calculation requires knowledge of the flow field inside the rotor. Moreover, those "flow efficiency" coefficients were introduced as an add-on to Cohen's equation, which was based on the assumption of a uniform flow

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Address correspondence to Oleg E. Aleksandrov, Department of Technical Physics, Ural Federal University (UPI), Mira 19, Ekaterinburg, 620002, Russian Federation. Tel.: +7 9122929147; Fax: +7 343 3754146. E-mail: aleks@dpt.ustu.ru

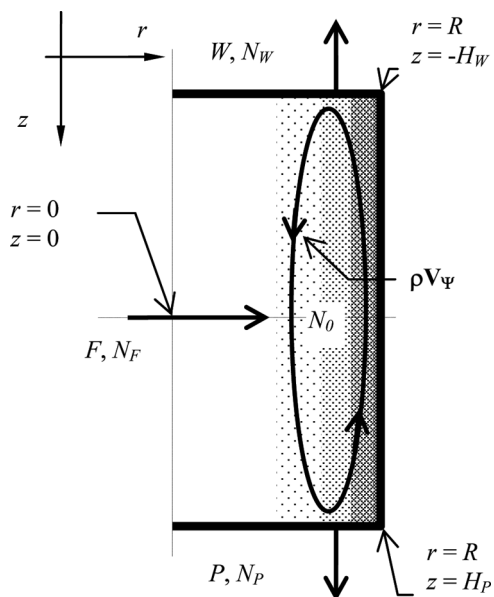


FIG. 1. A centrifuge schematic diagram.

within a rotor. Only there is still no answer to the question: what is the optimal circulation?

On the other hand, modern centrifuges demonstrate a performance capacity, which is 10–20% higher than predicted by the analytical Cohen's model (4). This gives grounds to consider the Cohen's model improper.

The results of this paper cannot be compared with the classical analytical or numerical expressions of for example (4), because the present work is not based on a solution of the Navier-Stokes equations and it does not calculate performance function representing the optimized separative power of a centrifuge for several simplified internal flow models. The question asked in the present work is: what is the best circulation? And the answer has been found out.

The axial nonuniformity of the internal circulation flow provides higher efficiency than the uniform flow of Cohen's model, more information can be obtained from (5,6). In the present paper, the problem of an energy efficient centrifuge is formulated for the first time. The paper suggests two analytical estimations for the maximum of the GC efficiency and, unlike the Dirac and works of later researchers, the present estimations directly connect the GC efficiency with the throughput and the circulation flow. The analytical estimations have been made for both: the ultimate upper limit of the GC separative power and the upper limit of the GC separative power per unit of energy consumption.

## THEORY

### Substantiation of a One-Dimensional Diffusion Equation

Cohen's method (4) reduces the problem of a two-dimensional distribution of the concentration within the

GC rotor to a one-dimensional equation for the concentration distribution along the GC axis. The advantage of this method is its simplicity and the opportunity to derive an analytical relation between the gas flow in the rotor and the GC efficiency. But in Cohen's original conclusion the gas flow structure within the rotor was significantly simplified:

1. It is assumed that the circulation flow intensity in the rotor did not vary along a GC and that flow closure occurred in negligibly small layers near the rotor faces.
2. The throughput is assumed to be negligibly small in comparison with the circulation flow within the rotor.

One is forced to believe that Cohen's one-dimensional diffusion equation is inapplicable to a quantitative description of the separation in GCs under the conditions of non-uniform circulation flow and finite throughput. For example, it is stated that any radial flow would reduce the GC efficiency because of convective mixing.

Let us suggest a better way to a one-dimensional diffusion equation, which is free from the above-mentioned limitations. The one-dimensional equation derivation being significant, the main steps are shortly repeated from (5,6). The mathematical procedure below seems complicated and difficult to follow, but this is the only way to prove the one-dimensional diffusion equation without unnecessary assumptions. The main purpose is to obtain a one-dimensional diffusion equation for an arbitrary internal circulation.

The flow inside the rotor (see Fig. 1) can be divided into two components:

1. the circulation and
2. the transit flow. Mathematically, this is an equivalent to a decomposition of the mass flux vector field into solenoidal (vortex) and irrotational (potential) parts. This is known as the Helmholtz decomposition. Such decomposition exists and is unique for any continuous vector field:

$$\rho \mathbf{V} = \rho \mathbf{V}_\Psi + \rho \mathbf{V}_\varphi = \rho \mathbf{Drot}(\Psi) + \mathbf{grad}(\varphi), \quad (3)$$

where  $\rho \mathbf{V}$  is the total mass flux vector;  $\rho \mathbf{V}_\Psi$  and  $\rho \mathbf{V}_\varphi$  are the vortex and potential mass flux vectors, respectively;  $\Psi = (0; \Psi; 0)$  is a vector potential of the circulation flow;  $\varphi$  is a scalar potential of the transit flow. The factor  $\rho D$  is introduced for convenience, and it is assumed that  $\rho D$  is constant. The single non-zero component of the vector  $\Psi$  is due to the two-dimensionality of the flow within the rotor (see Fig. 1).

Similar to the above, the isotope mass flux  $\Phi$  can be represented as

$$\Phi = \mathbf{rot}(\Psi_D) + \mathbf{grad}(\varphi_D). \quad (4)$$

The vector potential  $\Psi_D$  defines the vortex isotope mass flux and the scalar potential  $\varphi_D$  defines the potential isotope mass flux. On the other hand, by definition, the isotope mass flux  $\Phi$  is

$$\begin{aligned}\Phi_r &= -\rho D \cdot \left[ \frac{\partial N}{\partial r} + frN \cdot (1 - N) \right] + \rho V_r N, \\ \Phi_z &= -\rho D \frac{\partial N}{\partial z} + \rho V_z N,\end{aligned}\quad (5)$$

where  $N$  is the mass concentration of the isotope and  $f = (\Delta m \Omega^2)/(kT)$ .

It is possible to write Eq. (4) in the form

$$\Phi = \rho D \mathbf{rot}(\Psi N) + N_i \mathbf{grad}(\varphi), \quad (6)$$

where  $N_i$  is the isotope concentration at the point of the outgoing flux and  $i$  is an index equal to  $P$  for the enriching section of a GC and to  $W$  for the extracting one.

With expression (6), the diffusion equation

$$\text{div}(\Phi)_i = 0 \quad (7)$$

is satisfied identically, because of  $\text{div}(\Phi_i) = \rho D \text{div}(\mathbf{rot}(\Psi N)) + N_i \text{div}(\mathbf{grad}(\varphi))$ , but  $\text{div}(\mathbf{rot}(\mathbf{F})) = 0$  for any function  $\mathbf{F}$  and  $\text{div}(\mathbf{grad}(\varphi)) = \text{div}(\rho \mathbf{V}) - \rho D \text{div}(\mathbf{rot}(\Psi)) = \text{div}(\rho \mathbf{V}) = 0$  from definition (3). The boundary conditions for the isotope mass flux  $\Phi$  are satisfied identically, too. Therefore, expression (6) is the solution of the diffusion equation (7) for a GC.

Equating expressions (5) and (6) to each other allows to obtain

$$\begin{cases} -\left[ \frac{\partial N}{\partial r} + frN \cdot (1 - N) \right] + \frac{\rho V_{\varphi i}}{\rho D} (N - N_i) - \Psi \frac{\partial N}{\partial z} = 0 \\ -\frac{\partial N}{\partial z} + \frac{\rho V_{\varphi i}}{\rho D} (N - N_i) + \Psi \frac{\partial N}{\partial r} = 0 \end{cases} \quad (8)$$

this system of equations is identical to Eq (7). The following substitutions were made in (5):

$$\begin{aligned}\mathbf{rot}(\Psi_D) &= N \rho D \cdot \mathbf{rot}(\Psi) + \rho D \Psi \times \mathbf{grad}(N) \\ &= N \rho \mathbf{V}_\Psi + \rho D \Psi \times \mathbf{grad}(N).\end{aligned}\quad (9)$$

Eliminating the terms containing  $\partial N/\partial z$  from the first equation of Eq. (8) and the terms containing  $\partial N/\partial r$  from the second equation, we set

$$\begin{cases} -(1 + \psi^2) \frac{\partial N}{\partial r} - frN \cdot (1 - N) + v_r \cdot (N - N_i) = 0 \\ -(1 + \psi^2) \frac{\partial N}{\partial z} - fr\psi N \cdot (1 - N) + (v_z + \psi v_r) \cdot (N - N_i) = 0,\end{cases} \quad (9)$$

where  $v = \begin{pmatrix} v_r \\ 0 \\ v_z \end{pmatrix} = (\rho \mathbf{V}_\varphi / \rho D)$ . So far, no simplifications have been made and Eq. (9) is the exact equivalent of Eq. (7) without any assumptions.

The second equation in Eq. (9) can be subjected to a radial averaging under the conventional assumption that the concentration depends weakly on  $r$ , that is, by the substitution  $\langle N(r) \cdot X(r) \rangle \approx \langle N(r) \rangle \cdot \langle X(r) \rangle$ . The equation similar to that of Cohen's can be obtained:

$$\begin{aligned}-\frac{\partial N}{\partial z} - \frac{f \langle r \Psi \rangle}{\langle \Psi^2 \rangle} N \cdot (1 - N) + \frac{\langle v_z \rangle}{1 + \langle \Psi^2 \rangle} \left( 1 + \frac{\langle \Psi v_r \rangle}{v_z} \right) \\ \cdot (N - N_i) = 0,\end{aligned}\quad (10)$$

where  $\langle X \rangle = \frac{1}{\pi R^2} \int_0^R 2\pi r X(r) dr$ . Equation (10) differs from Cohen's original equation as it is applicable to an arbitrary distribution of the internal flow, that is, for any  $\Psi$ .

The radial velocity component  $v_r = v_r(z)$  related to the throughput depends on the feeding method. We will not elaborate on this point. Let us assume that the throughput does not have the radial velocity component ( $v_r = 0$ ). This will not affect the radial velocity related to the circulation. In this case, the term containing  $v_r$  vanishes from Eq. (10) and becomes entirely identical in the form to Cohen's equation.

To integrate Eq. (10), it is necessary either to know the circulation potential or to make an assumption concerning its form. Let us introduce the following assumption:  $\Psi(r, z) = \Psi_z(z) \Psi_r(r)$ . Then, the circulation amplitude (5) can be introduced in the form  $\Psi_0(z) = \sqrt{\langle \Psi_z^2(z) \rangle}$ , and the profile coefficient is

$$a = \frac{\langle \Psi_r \rangle}{\sqrt{\langle \Psi^2 \rangle}}, \quad (11)$$

where  $a$  is a value characterizing the flow profile along 0R axis and its maximum is  $a = R\sqrt{1/2}$  for the case of  $\Psi = \Psi_0(z) \cdot r$ .

Taking into account Eq. (11), Eq. (10) takes the following form

$$\begin{aligned}-\frac{d}{dz} \left( \ln \left( \frac{N}{1 - N} \right) \right) + \frac{fa \Psi_0(z)}{1 + \Psi_0(z)^2} \\ - \frac{\langle v_z \rangle}{1 + \Psi_0(z)^2} \frac{N - N_i}{N \cdot (1 - N)} = 0.\end{aligned}\quad (12)$$

Equation (12) should be linearized to be solved. Cohen linearized it on the assumption that the isotope concentration is small ( $N \ll 1$ ). A lot more general form can be obtained by requiring only the  $|N - N_i| \ll 1$  and taking into account that

$$\begin{aligned}
\ln(\alpha_i) &= \ln(1) + (N_i - N) \frac{d \ln(\alpha_i)}{dN_i} \Big|_{N_i=N} + O((N_i - N)^2) \\
&= (N_i - N) \frac{d \ln\left(\frac{1-N_i}{N_i}\right)}{dN_i} \Big|_{N_i=N} + O((N_i - N)^2) \\
&= -\frac{(N_i - N)}{N(1 - N)} + O((N_i - N)^2)
\end{aligned} \quad (13)$$

Hence,  $(N - N_i)/(N(1 - N))$  is the first term of the separation factor logarithm expansion into a series in  $N$  near of  $N_i$ . With the precision of the next series term  $\sim(N - N_i)^2$  one can write

$$-\frac{d}{dz}(\ln \alpha_i) + \frac{fa\Psi_0}{1 + \Psi_0^2} - \frac{v_z}{1 + \Psi_0^2} \ln \alpha_i = 0, \quad (14)$$

here are denoted  $\alpha_i = [N/(1 - N)][(1 - N_i)/N_i]$  (see Table) and  $v_z = \langle v_z \rangle$ . Equation (14) is a one-dimensional linear diffusion equation for a GC with an arbitrary circulation. Thus, this linear diffusion equation makes Eq. (14) applicable to a modern gas centrifuge machine, because  $|N - N_i| \ll 1$  can be provided with a high separation factor, for example,  $|N - N_i|$  tends to zero with  $N$  going to 1.

Equation (14) can be solved analytically in two cases:

1. when the circulation  $\Psi_0(z)$  does not depend on  $z$  and
2. for the case of optimal circulation.

For constant circulation flow  $\Psi_0$  along the  $z$ -axis (Cohen's case)

$$\begin{aligned}
\ln \chi &= \frac{\pi a \Psi_0 \rho D \Delta m \cdot (\Omega R)^2}{k T F \Theta \cdot (1 - \Theta)} \\
&\quad \left(1 - (1 - \Theta)e^{\Theta K H_P} - \Theta e^{(1 - \Theta) K H_W}\right)
\end{aligned} \quad (15)$$

where  $\ln \chi = \ln(\alpha_P) - \ln(\alpha_W)$  is the overall separation factor of a GC,  $K = -\frac{F}{\pi R^2 \rho D \cdot (1 + \Psi_0^2)}$ , and  $\Theta = P/F$  are the ratio of

the product/feed flows in a GC (the cut). A detailed analysis of the solution (15) is outside the scope of this article, but the solution is simpler than the original Cohen's one. Some further details can be found in (5,6).

### Optimization Problem Formulation

An optimization task can be set for the circulation flow in a centrifuge. In the case of an ideal cascade, the goal is the cascade with minimum power consumption for a given separation regime (for given  $F$ ,  $P$ ,  $N_F$ , and  $N_P$ ). And there are two ways to define the optimal circulation within the rotor:

1. the ultimate separative power maximization;
2. the minimization of the power used per unit of the separative power.

Hereinafter, these optimized centrifuges will be referred to as a perfect centrifuge type 1 (PC type 1) and a perfect centrifuge type 2 (PC type 2), respectively.

The costs of separation can be divided into two groups:

1. associated with the manufacture of centrifuges and their assembly into a cascade, etc.
2. related to the operation of a centrifuge, i.e., electricity supply and so on.

If dominated by the first type of costs, the "economic" optimization coincides with the case of "the maximum separative power." If, however, dominated by the second type of costs, the optimization of "the maximum separative power per unit of energy used" is more interesting.

The only internal parameter in the proposed centrifuge model (14) is the circulation potential  $\Psi_0(z)$ . It is necessary to define  $\Psi_0(z)$  in such a way for the optimization goal to be obtained.

## RESULTS AND DISCUSSION

### Ultimate Separative Power Maximum

In the first case of optimization, for a given throughput  $F$  and the cut  $\Theta$ , the separative power is (1)

$$\delta U = \Theta \cdot (1 - \Theta) F \cdot (\ln \chi)^2 \quad (16)$$

and it will be a maximum along with the maximum of the overall separation factor  $\chi$ , where  $\ln \chi = \ln \alpha_P - \ln \alpha_W$ . A maximum of  $\ln \chi$  is achieved if  $d(\ln \alpha_i)/dz$  is a maximum at every point on the  $z$ -axis. The condition for the extremum is  $\frac{d}{d\Psi_0} \left( \frac{d}{dz}(\ln \alpha_i) \right) = 0$  or  $\frac{d}{d\Psi_0} \left( \frac{fa\Psi_0 - v_z \ln \alpha_i}{1 + \Psi_0^2} \right) = 0$ . In turn,

$$\begin{aligned}
\frac{d}{d\Psi_0} \left( \frac{fa\Psi_0 - v_z \ln \alpha_i}{1 + \Psi_0^2} \right) &= \frac{\partial}{\partial \Psi_0} \left( \frac{fa\Psi_0 - v_z \ln \alpha_i}{1 + \Psi_0^2} \right) \\
&+ \frac{\partial}{\partial \ln \alpha_i} \left( \frac{fa\Psi_0 - v_z \ln \alpha_i}{1 + \Psi_0^2} \right) \cdot \frac{d \ln \alpha_i}{d\Psi_0}.
\end{aligned}$$

**Table**  
**Denotations**

$\chi = [N_P/(1 - N_P)]$ $[(1 - N_W)/N_W]$	the overall separation factor of a GC
$\alpha_i = [N/(1 - N)]$ $[(1 - N_i)/N_i]$	the separation factor for the section $i$ of GC ( $i = P$ or $W$ )
$\alpha_P = [N/(1 - N)]$ $[(1 - N_P)/N_P]$	the separation factor for the enriching section of GC
$\alpha_W = [N/(1 - N)]$ $[(1 - N_W)/N_W]$	the separation factor for the extracting section of GC
$\ln(\alpha_i)$	the separation factor logarithm for the section $i$ of GC
$\frac{\ln(\alpha_i)}{faH_i}$	the normalized separation factor logarithm for the section $i$ of GC



However, we should assume  $\frac{d \ln \alpha_i}{d \Psi_0} = 0$ , as  $\ln \alpha_i$  is the extreme separation factor. Finally,

$$\frac{fa \cdot (1 - \Psi_0^2) + 2v_z \Psi_0 \ln \alpha_i}{(1 + \Psi_0^2)^2} = 0. \quad (17)$$

From Eq. (17) it follows that

$$\Psi_0 = \frac{v_z \ln \alpha_i}{fa} + \sqrt{\left(\frac{v_z \ln \alpha_i}{fa}\right)^2 + 1}. \quad (18)$$

Let us note that  $\ln \alpha_P(z=0) = \ln \alpha_P$  and  $\ln \alpha_P(z=H_P) = 0$ . Consequently, the optimal  $\Psi_0(z)$  has a maximum at the feeding point and decreases down to  $\Psi_0=1$  at the rotor ends ( $z=H_P$  or  $z=H_W$ ). Since the circulation flow should be continuous on the boundary between the enriching and extracting sections, the centrifuge optimization with respect to the circulation flow sets up one more relation:  $\frac{\Theta}{1-\Theta} = \frac{\ln \alpha_W}{\ln \alpha_P}$ , thus, with  $H_P=H_W$ , it is necessary that  $\Theta=0.5$ .

Back substituting expression (18) into Eq. (14) gives the main relation for the PC type 1:

$$\frac{d \ln \alpha_i}{dz} = \frac{fa}{2\Psi_{0opt}(z)}, \quad (19)$$

where  $\Psi_{0opt}$  is the optimal profile of the internal circulation.

Equation (19) can be integrated by introducing the denotations

$$\begin{aligned} \beta_i &= \frac{v_{zi} \ln \alpha_i}{fa}; \\ l_i &= H_i \cdot v_{zi}; \\ l &= z \cdot v_{zi}; \end{aligned} \quad (20)$$

the solution of Eq. (19) can be represented in the form:

$$l_i = \beta_i \cdot \left( \sqrt{1 + \beta_i^2} + \beta_i \right) + \ln \left( \sqrt{1 + \beta_i^2} + \beta_i \right). \quad (21)$$

It is impossible to derive an analytical expression for  $\beta_i(l_i)$  from function (21).

The optimal circulation potential (18) can be written in the same notation:

$$\Psi_0 = \sqrt{1 + \beta_i^2} + \beta_i \text{ or } \beta_i = \frac{\Psi_0^2 - 1}{2\Psi_0} \quad (22)$$

The relation  $\frac{\beta_i}{l_i} = \frac{\ln(\alpha_i)}{faH_i}$  can be considered as the normalized separation factor per unit length of the centrifuge

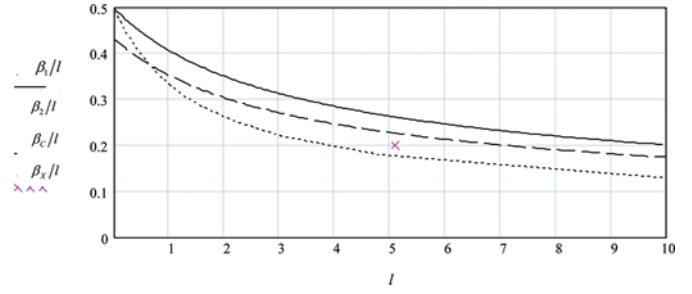


FIG. 2. The normalized ultimate separation factor per the rotor unit length as a function of the normalized rotor length. 1 — is PC type 1, 2 — is PC type 2; C - - is optimized Cohen's centrifuge; X is the real centrifuge point. (Color figure available online)

rotor, and value  $l_i$  can be considered as the effective rotor length of a GC. It should be stressed that value  $l_i$  depends upon  $H_i$  (rotor length) and  $v_{zi}$  (throughput). For the PC type 1, an ideal dependence of the separation factor on the throughput and the centrifuge length is shown in Fig. 2. The optimized Cohen's centrifuge curve and the point of a real centrifuge can be found there, too. A data point of any real centrifuge cannot occur above the curve presented in the given figure. Figure 2 shows that an increase either in the rotor length ( $H$ ) or the throughput ( $v_z$ ) reduces the maximal separation factor per unit length of the PC type 1.

The separative power of a PC type 1 is the ultimate upper limit for the separative power of any GC as the function of the GC external parameters and it can be written as

$$\delta U_{\max} = 2 \frac{\beta_i^2}{l_i} \delta U_D.$$

Figure 3 shows the reachable part of Dirac's separative power (2) as the function of the rotor length and the throughput. A GC that can exceed this limit does not exist. It demonstrates that Dirac's limit of a GC separative power can be reached only with infinitely large  $l$  ( $l \rightarrow \infty$ ), which leads to an infinitely large throughput and circulation flow. Existing GCs cannot reach the Dirac's limit.

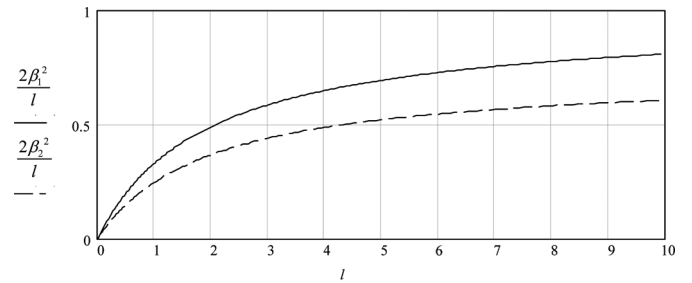


FIG. 3. The achievable part of Dirac's separative power as the function of the normalized rotor length. 1 — is PC type 1, 2 — is PC type 2.

### Minimization of the Power Used

With other case of the optimization (PC type 2), the minimization of energy consumption per unit of the separative power is the intention.

An opinion exists that only a small fraction of the energy consumed by a GC is used to maintain the internal circulation. But it is not correct. If a GC works without any feed and circulation, the energy consumption tends to zero because of the negligible bearing friction. The main energy losses arise due to a viscous friction within gas. The viscous friction losses are accounted for by three mechanisms:

1. the feed gas needs to be accelerated to reach the gas rotation velocity,
2. the gas withdrawal by product and waste scoops leads to an interaction of the fixed scoops with high-speed gas,
3. the internal circulation results in viscous dissipation of energy.

The first two seem to have no connection with the internal circulation intensity. But one should take into account that the second loss mechanism is not only a mechanism of the gas withdrawal, but it is the main mechanism to provide circulation; and for the greater circulation flow, the greater scoops interaction with gas is required. For the first loss mechanism, as it was shown earlier (see Eq. 22), the internal circulation flow and the throughput are connected and the optimal circulation flow is approximately proportional to the throughput. The further inquiry will lead us to the similar proportionality in the case of the minimization of energy consumption per unit of the separative power.

Thus, all significant mechanisms of GC energy consumption seem to be proportional to the internal circulation flow intensity. Moreover, the internal circulation of direct viscous dissipation is not so small. In fact, the internal circulation of direct viscous dissipation is about the Navier-Stokes equation member  $(\mathbf{v} \cdot \nabla) \mathbf{v}$  or about  $R\Omega V_\Psi$ , its value is at least comparable to the first loss mechanism.

Unfortunately, there is no public data about any actual energy consumption distribution in a centrifuge between those loss mechanisms. In some publications (7) the estimated energy losses connected with the internal circulation are dominant in the total GC energy consumption.

So far, the main GC energy consumption is due to the need to maintain the circulation within a rotor. It is obvious, that such energy usage should be proportional to the circulation potential amplitude  $\Psi_0(z)$ .

Thus, it is essential to demand the maximum of  $\frac{\delta U}{\Psi_0} = \Theta \cdot (1 - \Theta) F \cdot \frac{(\ln \chi)^2}{\Psi_0}$  (see Eq. 16) or the maximum of  $\frac{\ln \chi}{\sqrt{\Psi_0}}$ . This can be reached for maximum of  $\frac{1}{\sqrt{\Psi_0}} \frac{d}{dz} (\ln \alpha_i)$  at each point along GC rotor axis. Dividing Eq. (14) by

$\sqrt{\Psi_0}$ , we can obtain

$$\frac{1}{\sqrt{\Psi_0}} \frac{d}{dz} (\ln \alpha_i) = \frac{fa\sqrt{\Psi_0}}{1 + \Psi_0^2} - \frac{v_z}{1 + \Psi_0^2} \frac{\ln \alpha_i}{\sqrt{\Psi_0}}. \quad (23)$$

The differentiating of Eq. (23) on  $\Psi_0$  produces

$$\frac{\partial}{\partial \Psi_0} \left( \frac{1}{\sqrt{\Psi_0}} \frac{d}{dz} (\ln \alpha_i) \right) = \frac{fa\sqrt{\Psi_0}}{(1 + \Psi_0^2)} \left( \frac{1}{2\Psi_0} - \frac{2\Psi_0}{(1 + \Psi_0^2)^2} + \frac{2}{(1 + \Psi_0^2)} \frac{v_z \ln \alpha_i}{fa} \right). \quad (24)$$

With the denotations (20), we can set the following equation to determine the PC type 2 optimal circulation from Eq. (24)

$$\frac{1}{2\Psi_0} - \frac{2\Psi_0}{(1 + \Psi_0^2)^2} + \frac{2}{(1 + \Psi_0^2)} \beta_i = 0. \quad (25)$$

Equation (25) connects an optimal  $\beta_i$  and  $\Psi_0$ . The expression for the optimal circulation potential for PC type 2 is

$$\Psi_0 = \frac{1}{3} \left( 2\beta_i + \sqrt{4\beta_i^2 + 3} \right), \quad (26)$$

as it is followed from Eq. (25). Or the equivalent of the expression (26) is

$$\beta_i = \frac{3\Psi_0^2 - 1}{4\Psi_0}. \quad (27)$$

Back substituting expression (27) into Eq. (14) gives the relation for the PC type 2:

$$\frac{d\Psi_0}{dl} = \frac{\Psi_0}{(3\Psi_0^2 + 1)}. \quad (28)$$

Equation (28) can be integrated as

$$l = \frac{3}{2} \Psi_0^2 + \ln(\Psi_0) + const. \quad (29)$$

The constant in Eq. (29) can be found from boundary conditions. Keeping in mind that at the rotor ends  $l=0$  and  $\beta_i(l=0)=0$ . Hence, from Eq. (27) it follows that  $\Psi_0(l=0)=1/\sqrt{3}$ . Therefore,  $const = \frac{1}{2}(\ln(3) - 1)$ . The final expression for  $l(\Psi_0)$  is

$$l = \frac{3}{2} \Psi_0^2 + \ln(\sqrt{3}\Psi_0) - \frac{1}{2} \quad (30)$$

By substitution of Eq. (26) into Eq. (30), the expression for  $l(\beta_i)$  can be obtained, if it is necessary.

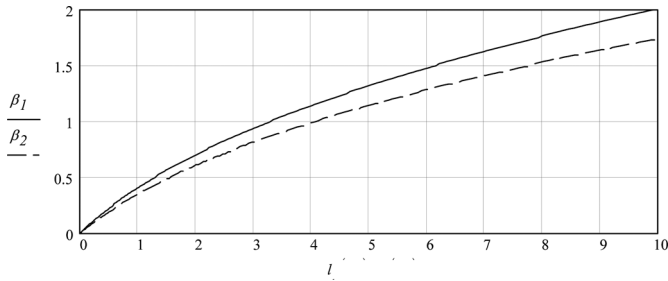


FIG. 4. The normalized optimal separation factor  $\beta(l)$  as function of the normalized rotor length. 1 — is PC type 1; 2 — is PC type 2.

It is interesting to compare the separation factor dependence on two types of PCs: Eq. (22) and Eq. (27). When,  $\beta_1(l=0)=0$  the circulation potential  $\Psi_0(l=0)=1$  for the PC type 1. This means that for the PC type 1 the radial circulation flow at the rotor face has finite quantity. On the other hand, for the PC type 2  $\Psi_0(l=0)=1/\sqrt{3}$ , which means that the radial circulation flow has to be less at the rotor face.

The normalized ultimate separation factor per the GC rotor unit length as a function of the normalized rotor length is shown in Fig. 2. Both PCs have no optimal length and the maximal separation factor per rotor length unit is steadily decreasing.

The reachable fraction of the Dirac's separative power for the PC type 2 is shown in Fig. 3. It should be mentioned that the separative power of the PC type 2 is less than the one of the PC type 1. Moreover, the PC type 2 cannot reach the Dirac's separative power limit at all.

To make it clearer, Fig. 4 and Fig. 5 show  $\beta(l)$  and the optimal circulation flow profile  $\Psi_0(l)$  for both types of a GC. This flow profile is the guiding line for any numerical or experimental GC internal flow optimization. The obtained circulation potential directly provides the optimal axial dependence on the circulation intensity  $\rho V_z(l)$ . The amplitude of  $\Psi_0(l)$  can be interpreted as the total mass flux along the rotor axis  $0.5 \int_0^R |\rho V_z(r)| r dr$ .

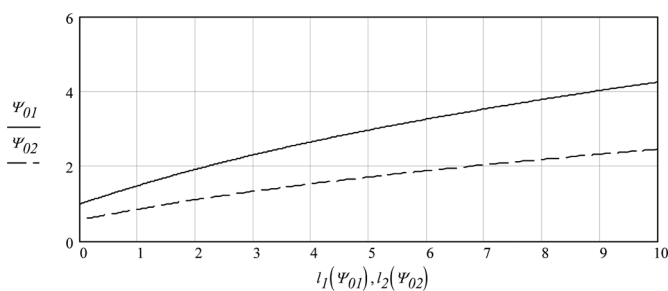


FIG. 5. The optimal circulation potential  $\Psi_0(l)$  pprofile as function of the normalized rotor length. The point  $l=0$  corresponds to a rotor cap and the point  $l>0$  corresponds to feed point. 1 — is PC type 1; 2 — is PC type 2.

It is obvious that the separation factor, the separative power, and the circulation flow intensity are smaller for the PC type 2 than the PC type 1. But the separation factor and the separative power are less decreasing in comparison with the circulation flow intensity.

## CONCLUSIONS

The radial averaging method is applicable, without sacrificing accuracy, when describing the separation in GCs with an essentially nonuniform distribution of the circulation flow along the rotor length.

The radial mass fluxes in GCs do not inadvertently cause a decrease in the separative power. Moreover, a GC with the nonuniform axial circulation flow has more separative power than the one with the uniform axial circulation flow.

There is the optimal internal circulation flow in a GC. The ideal distribution of the mass flux along the rotor axis can be compared with the result of numerical calculations of the GC internal flow, and this can help improve the internal circulation.

The absolute upper limit for the separative power of a gas centrifuge is calculated as an analytic function of the centrifuge parameters. The main difference from previous estimates, that is, those of Dirac's, is that for the first time a value of the throughput of a gas centrifuge has been taken into account as a parameter and an exact expression without any unknowns has been obtained. These functions allow us to compare the efficiency of any existing GC with the ultimate limit on the basis of the well-known external GC parameters.

For the first time the optimization of an energy-efficient centrifuge has been examined and the maximum estimate for the separative power has been calculated. The comparison of the separation characteristics for the absolute upper limit of the separative power and the separative power of an energy-efficient centrifuge has been conducted. The energy-efficient centrifuge will consume less energy per unit of the separative power. This ideal model allows us to determine the low limits of energy consumption per unit of the separative power.

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